Simulated Holograms

A Simple Introduction to Holography

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Holography is a well-known method of generating three-dimensional pictures, and even students are accustomed to this new imaging process. Influenced by their experiences and observations in connection with holograms, young people are very interested in the physical background of holography. Therefore we were looking for a method of teaching holography from a very elementary point of view.

The main idea of this project is to generate holograms of simple object structures, i.e., one, two, three,...etc. point sources arranged in a plane or in space, which provide a step-by-step introduction to the holographic method. Unfortunately, the production of holograms for such simple object structures is rather difficult, especially if only standard demonstration equipment is available. Furthermore, teaching the holographic recording and reconstruction process requires prerequisites that generally are not given in elementary physics courses. To overcome these difficulties we use a computer and a dot matrix printer to simulate the holographic recording process.

Production of Simulated Holograms

Step 1: Superposition of waves

As our first step we calculate the superposition S of a plane reference wave and the respective spherical object wave(s) in the hologram plane (see Fig. 1). For the sake of simplicity we use the following approximations and assumptions:

- From the phasor diagram in Fig. 2 for the superposition S of the plane reference wave and one spherical object wave with amplitudes A_0 and A_1 respectively, it is evident that for $A_0 \gg A_1$ the magnitude of S can be expressed as the projection of S onto A_0. In practice we found that $A_0 / A_1 \geq 3$ is already a good approximation.
- The amplitude of the spherical object wave depends on r, the distance between the point source Z and the hologram point $P(x,y)$ (see Fig. 1). We assume the amplitude to be constant in the hologram plane, which is justified for r larger than the hologram dimensions.
- We neglect the time dependence in the wave equation and choose the phase of the reference wave such that its amplitude is maximal in the hologram plane.

Considering these approximations and assumptions, the superposition S, e.g., of a plane and a spherical wave at the point $P(x,y)$ in the hologram plane (see Fig. 1) can be written in the rather simple form:

$$S(x,y) = A_0 + A_1 \cos(2\pi r_1/\lambda) \quad (1)$$

where λ is the wavelength and $r_1 = (a^2 + x^2 + y^2)^{1/2}$. The object wave is starting at $Z(0,0,-a)$.

For more point sources arranged somehow in space, the respective superposition is obtained in a similar way. For example, in the case of two point sources at $Z_1(x_0,0,-a)$ and $Z_2(-x_0,0,-a)$ in the x, y-plane parallel to the hologram plane, the superposition is given by:

$$S(x,y) = A_0 + A_1 \cos(2\pi r_1/\lambda) + A_2 \cos(2\pi r_2/\lambda) \quad (2)$$

with $r_1 = [a^2 + (x + x_0)^2 + y^2]^{1/2}$, $r_2 = [a^2 + (x - x_0)^2 + y^2]^{1/2}$, and x_0 being the displacement in x-direction.

Step 2: Representing S on the monitor screen

$S(x,y)$ is a continuously varying function that cannot be represented on the monitor screen or on the printer sheet in a direct way because in both cases only a "digital" representation is possible: either a dot is printed or it is not. To overcome this difficulty we describe $S(x,y)$ by the respective dot density. For this purpose the number of dots per unit area is chosen proportional to S. In addition the dots have to be randomly distributed in order to avoid additional regular structures in the simulated holograms.

To fulfill these two conditions, we choose the probability of printing a dot proportional to $S(x,y)$ [for $A_0 = 3$ and $A_1 = 1$ in Eq. (1), for example, the respective probability is $S/4$] and a dot is printed when a random number between 0 and 1 is smaller than the respective normalized value of S as indicated in the following BASIC program line:

```
IF RANDOM < S(x,y)/4 THEN PLOT P(x,y).
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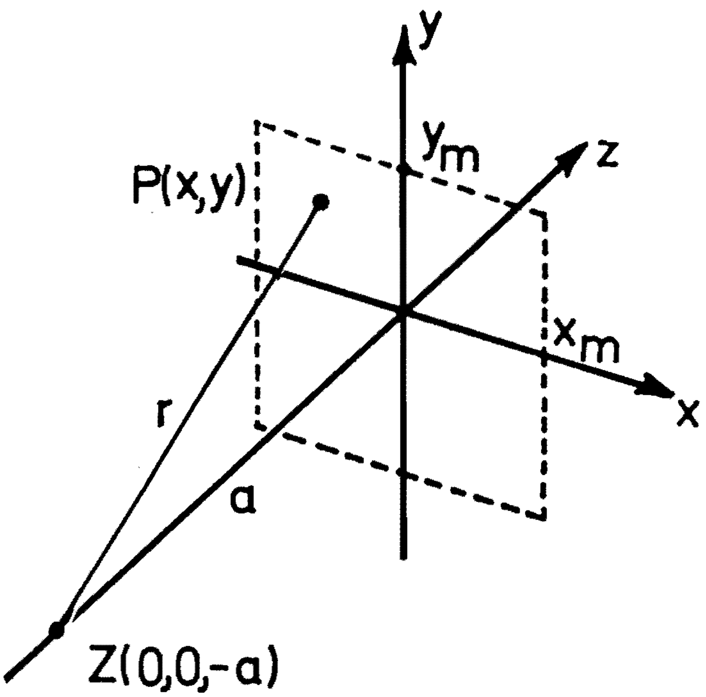

Fig. 1 Geometry for the simulation. *Z*(0,0,–*a*): point source located on the z-axis; *P* (*x*,*y*): point in the hologram plane; *r*: distance between *Z* and *P*.

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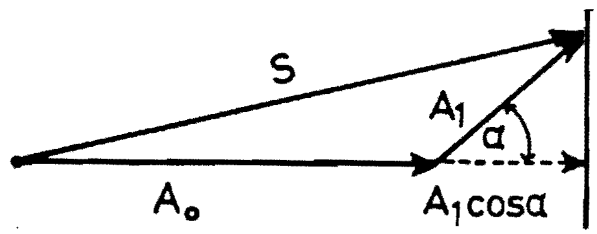

Fig. 2 Phasor diagram representing the superposition S of two waves with the amplitudes A_0 and A_1 and the phase shift α.

For $S(x,y)/4 \approx 1$ the probability of printing a dot is rather high, but it is still possible that the respective random number is larger than $S/4$ and that the dot is not printed. This apparent error is compensated over a larger area. It can be proven that the number of dots per unit area is proportional to S.

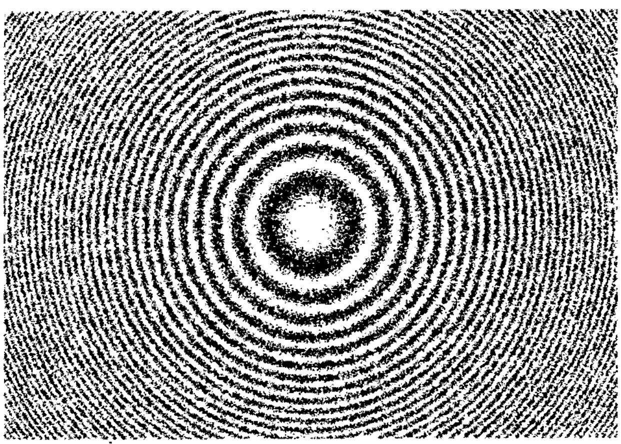

Fig. 3a.

Fig. 3 Results of the simulation for some representative point source configurations (plane reference wave). (a) One point source at a finite distance to the hologram plane. (b) Two point sources located on a plane parallel to the hologram plane at a finite distance to the hologram plane. (c) Two point sources lying on a straight line normal to the hologram plane at different distances to the hologram plane. (d) Three-dimensional arrangement of four point sources (tetrahedron). (e) One point source at infinity.

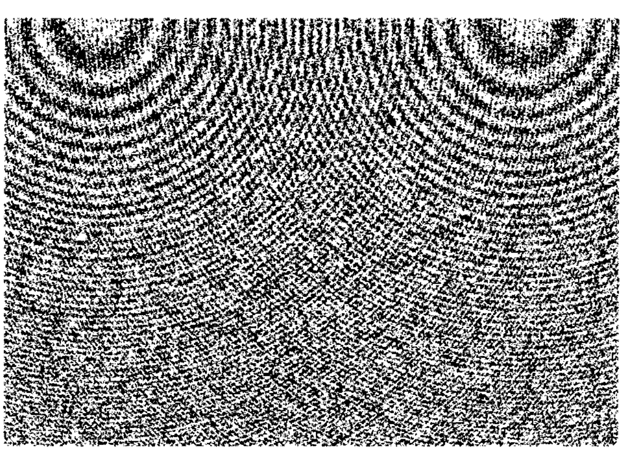

Fig. 3b.

During our calculations all lengths are expressed in an arbitrary unit LU, which is determined by the minimum dot separation; LU $\approx$ 0.3 mm for our printer. Good results are obtained for $a \approx 5000$ LU, $\lambda \approx 2$ LU, and $x_0 \approx 100$ LU.

Figure 3 represents the results of the simulation for some representative examples. In all cases we assumed a plane reference wave. Figure 3a shows the result for one point

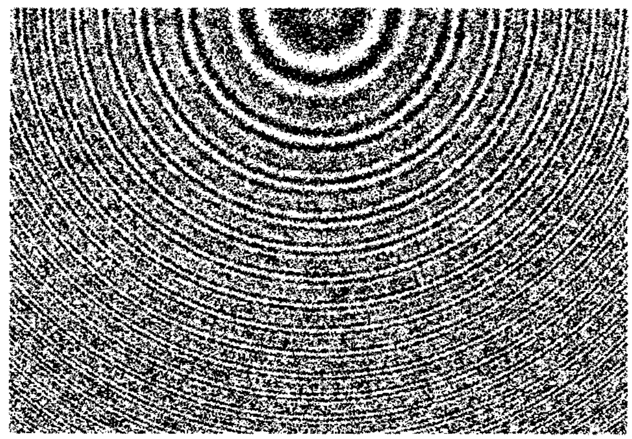

Fig. 3c.

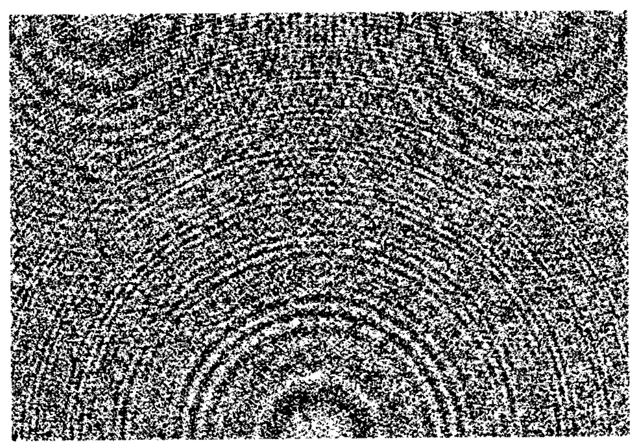

Fig. 3d.

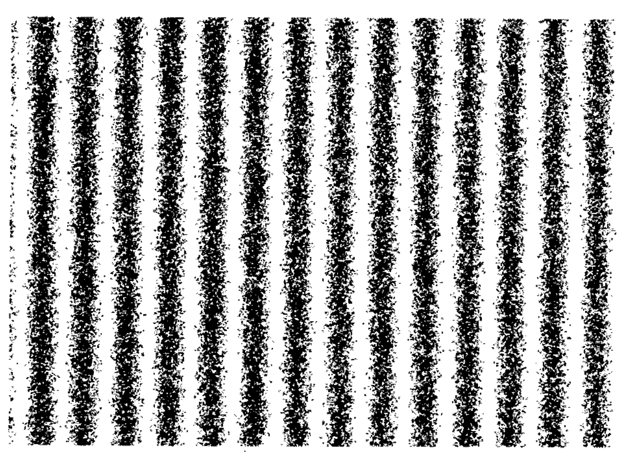

Fig. 3e.

source (spherical object wave). This pattern (Fresnel zone structure) is the basis of all holograms.[1-3] It differs from conventional Fresnel zone plates only in having gradual transitions between dark and light zones instead of abrupt changes. It proves that the obtained intensity distribution is sinusoidal, which is important for the reconstruction process (see Step 4).

In the following examples the calculation is restricted to a part of the hologram plane in order to get higher resolutions.

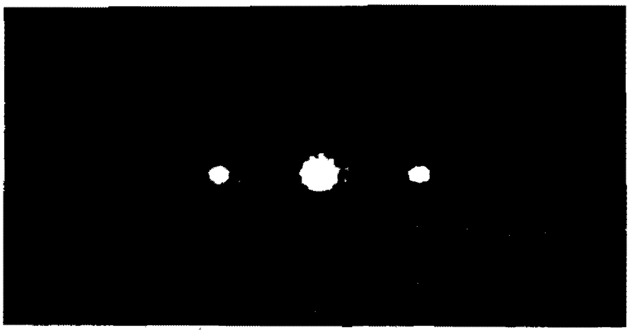

Fig. 4a.

Fig. 4. Representative examples of the reconstructed images in the case of: (a) The grating pattern in Fig. 3e (point source at infinity). (b) and (c) The pattern corresponding to a tetrahedron in Fig. 3d for different focusing and viewing conditions. Since the "grating-like" structure of the holograms is sinusoidal, besides the zero order only the first diffraction order appears during the reconstruction. The bright spot in the center belongs to the zero diffraction order and corresponds to the point source at infinity of the plane reference wave. In Fig. 4a we focused on infinity, therefore the images of the point sources placed at infinity are in focus. In Figs. 4b and c we focused on the tetrahedron arranged at a finite distance to the hologram plane. In the +1st diffraction on the right side of the central spot the image of the tetrahedron is in focus and on the left side, in the -1st diffraction order, it is out of focus and is responsible for the speckle structure. The central spot—now out of focus—appears in an enlarged form.

Fig. 4b.

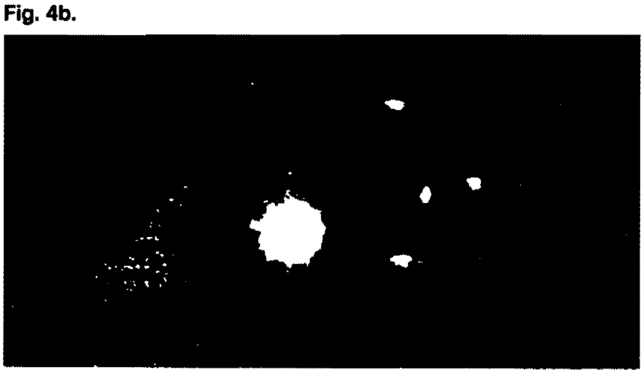

Fig. 4c.

Figure 3b illustrates the result for two point sources located in a plane parallel to the hologram plane. It shows two overlapping Fresnel zone structures. Figure 3c represents the result for two point sources located on a straight line vertical to the hologram plane at different distances. The respective pattern is a system of two concentric ring structures. Figure 3d shows the pattern in the case of a simple three-dimensional object. It consists of four point sources arranged like a tetrahedron—three points in a plane parallel to the hologram plane and one outside of this plane. With the knowledge of the pattern in Figs. 3a to 3c it is possible to recognize the respective arrangement of the points from the pattern itself. Figure 3e illustrates the result in the case of one point source located at infinity (one plane object wave). The result is a grating structure.

For an introductory course, other point-source configurations are also useful. Even the dependence on the amplitude of the object waves can be simulated. The superposition of a cylindric object wave and the plane reference wave is helpful to explain the transition from a grating to a Fresnel zone structure.

The examples in Fig. 3 are obtained with the screen resolution of about 600 × 400 dots. This resolution is not sufficient to get good results after minification, but it is more suited for reproduction. Good holograms were obtained for a resolution of about 3600 × 2400 dots. For this purpose we split the chosen hologram sector into 36 parts, calculated the superposition of the waves corresponding to the respective point source configuration for each part, made a hardcopy of the screen, and finally glued the sheets in the right order to get a large pattern of about 1.2 × 0.8 m with approximately 10^7 dots.

Step 3: Photographic minification

The large patterns have to be reduced photographically in order to get transparencies that then behave like any other optical hologram. For this purpose we used an ordinary amateur camera (film format 24 × 36 mm) and a high resolution black-and-white negative film (e.g., Agfapan professional 25 ASA, resolution 350 lines/mm).

The exposure and the development time of the film was chosen so that the photographic process is approximately "digital" in order to get a point-by-point recording, i.e., each dot gives a well-separated "hole" in the film emulsion. We found the reduction 1 : 30 to 1 : 50 sufficient to get structures that are still resolved on the film and already show diffraction for light waves.

Step 4: The reconstruction

Holograms have many properties of a grating, and so the reconstruction of the hologram images is similar to a diffraction experiment with a grating. The experimental setup is described in many optics textbooks, including Hecht's.[2]

Figure 4 illustrates through photographs the results of the reconstruction for some examples. Figure 4a represents the reconstructed image of the hologram pattern from Fig. 3e, and shows the surprising effect that only the zero and first refraction orders appear. This proves that the simulation process generates a sinusoidal transmission structure of the transparency.[2] This result is important from the technical point of view and it simplifies application of the simulated holograms in the teaching process. Furthermore it indicates the possibility of producing sinusoidal diffraction structures in a very simple way.

From the origin of the pattern in Fig. 3e (single point source at infinity) one expects only one point at infinity, but the reconstruction delivers two points—one at "plus" and one at "minus" infinity. This ambiguity is due to the neglected time dependence in the wave description. For example, in Eq. (1) it is impossible to distinguish between "incoming" and "outgoing" waves. Therefore two images appear in the reconstruction—a real and a virtual (conjugate) image.[2-4]

Figure 4b shows the reconstruction for the hologram pattern in Fig. 3d (three-dimensional point arrangement, tetrahedron). The three-dimensional character is demonstrated by focusing on the top of the tetrahedron. The other three point sources originally arranged in a plane parallel to the hologram plane are out of focus, as expected. In Fig. 4c we have chosen another camera position and another diaphragm. The view of the tetrahedron has changed. This change of the perspective is another test for the three-dimensional character of the reconstructed images. In addition all points of the tetrahedron appear approximately in focus on this photo. This effect indicates that the depth of focus increases when the diameter of the diaphragm decreases, an observation well known from real holograms.[2]

Concluding Remarks

The simulation of holograms for simple point source arrangements allows a step-by-step introduction to holography and gives a deeper insight into the holographic recording and reconstruction process. No special prerequisites are necessary. The method only requires the mathematical description of waves in its simplest form. The applied computer program is rather simple (some BASIC lines). Also, it is possible to explain the holographic imaging process from a very elementary point of view, based on Huygen's principle alone:

> Every simulated hologram represents a characteristic dot distribution that is transformed into a respective hole distribution through the photographic minification process. This property allows the application of Huygens' principle: the dots represent "frozen" single point sources and their distribution represents the frozen wavefront. The holes on the transparency can be regarded as single point sources that can be reactivated. When light shines on the transparency, each hole becomes a source of spherical wavelets and the envelope of these wavelets represents the reactivated wavefront just behind the hologram plane.

Unfortunately the simulation is limited to a small number of point sources. Otherwise the calculation time becomes too large. For more complicated object structures, some proposed computer-generated holograms[3,4] are more convenient, but less suited for an introductory course.

References

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2. E. Hecht, *Optics* (Addison Wesley, Reading, MA, 1987).
3. Wai-Hon Lee, "Computer generated holograms: Techniques and applications" in E. Wolf, *Progress in Optics XVI* (North-Holland Publishing, Amsterdam, 1978).
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Erratum

In *TPT's* February "AstroNotes" (page 93) a line of type was inadvertently dropped. The sentence that begins at the bottom of the first column should read, "The value of density I used was the actual count at the shortest distance counted divided by πd^2, assuming the space is locally flat." Sorry about that.